



PERFORMANCE ANALYSIS OF LINEAR AND NONLINEAR RESOURCE ALLOCATION TECHNIQUES IN OFDM SYSTEM

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Abstract— Multiuser orthogonal frequency division multiplexing (MU-OFDM) is a promising technique for achieving high downlink capacities in future cellular and wireless LAN systems. OFDM base stations allow multiple users to transmit simultaneously on different subcarriers during the same symbol period. The sum capacity of MU-OFDM is maximized when each subchannel is assigned to the user with the best channel-to-noise ratio for that subchannel. In this paper, we focus on the base station resource allocation in terms of subcarrier and power to each user to maximize the sum of user data rates subject to constraints on total power, bit error rate, and proportionality among user data rates. There are several methods proposed in the literature which are being iterative non-linear methods which are suitable for offline optimization in the special case of sub-channel SNR and iterative route finding method has linear time complexity in the number of users and $N \log N$ complexity in the number of subchannels. The proposed method is a low-complexity method which works under varying the restriction of high subchannel SNR and yields higher user data rates. It is also shown that with the proposed resource allocation algorithm, sum capacity is distributed more fairly and flexibly among users than the sum capacity maximization method.

Keywords- MU-OFDMA, SNR, Resource Allocation

I. INTRODUCTION

OFDMA, also referred to as Multiuser-OFDM

is being considered as a modulation and multiple access method for 4th generation wireless networks. OFDMA is an extension of Orthogonal Frequency Division Multiplexing (OFDM), which is currently the modulation of choice for high speed data access systems such as IEEE 802.11a/g wireless LAN and IEEE 802.16a fixed wireless broadband access systems.

Orthogonal frequency division multiplexing (OFDM) is a promising technique for the next generation of wireless communication systems.

OFDM divides the available bandwidth into N orthogonal subchannels. By adding a cyclic prefix (CP) to each OFDM symbol, the channel appears to be circular if the CP length is longer than the channel length. Each subchannel thus can be modeled as a time-varying gain plus additive white Gaussian noise (AWGN). Besides the improved immunity to fast fading brought by the multicarrier property of OFDM systems, multiple access is also possible because the subchannels are orthogonal to each other.

OFDM adds multiple access to OFDM by allowing a number of users to share an OFDM symbol. Two classes of resource allocation schemes exist: fixed resource allocation and dynamic resource allocation [1]. Fixed resource allocation

schemes, such as Time Division Multiple Access (TDMA) and Frequency Division Multiple Access (FDMA), assign an independent dimension, e.g. time slot or subchannel to each user. A fixed resource allocation scheme is not optimal since the scheme is fixed regardless of the current channel condition. On the other hand, dynamic resource allocation allocates a dimension adaptively to

the users based on their channel gains. Due to the time-varying nature of the wireless channel, dynamic resource allocation makes full use of multiuser diversity to achieve higher performance.

The problem of assigning subcarriers and power to the different users in an OFDMA system has recently been an area of active research. In the margin-adaptive resource allocation problem was tackled, wherein an iterative subcarrier and power allocation algorithm was proposed to minimize the total transmit power given a set of fixed user data rates and bit error rate (BER) requirements. In the rate-adaptive problem was investigated, wherein the objective was to maximize the total data rate over all users subject to power and BER constraints. It was shown in that in order to maximize the total capacity, each subcarrier should be allocated to the user with the best gain on it, and the power should be allocated using the water-filling algorithm across the subcarriers. However, no fairness among the users was considered in this problem was partially addressed by ensuring that each user would be able to transmit at a minimum rate, and also in by incorporating a notion of fairness in the resource allocation through maximizing the minimum user's data rate. In the fairness was extended to incorporate varying priorities. Instead of maximizing the minimum user's capacity, the total capacity was maximized subject to user rate proportionality constraints. This is very useful for service level differentiation, which allows for flexible billing mechanisms for different classes of users. However, the algorithm proposed in involves solving non-linear equations, which requires computationally expensive iterative operations and is thus not suitable for a cost-effective real-time implementation.

This paper extends the work in by developing a sub-carrier allocation scheme that linearizes the power allocation problem while achieving approximate rate proportionality. The resulting power allocation problem is thus reduced to a solution to simultaneous linear equations. In simulation, the proposed algorithm achieves a total capacity that is consistently higher than the previous work, requires significantly less

computation, while achieving acceptable rate proportionality.

II. SYSTEM MODEL

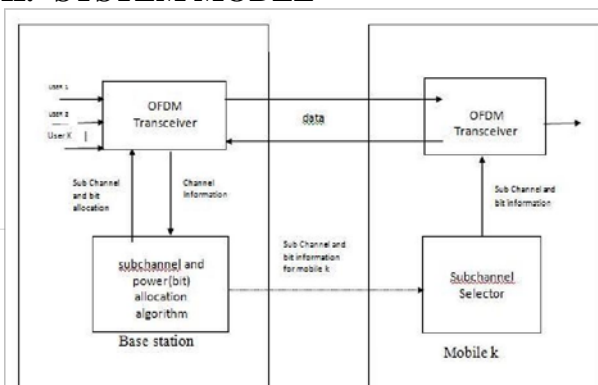


Fig. 1: Multiuser OFDM system Block Diagram

A multiuser OFDM system is shown in Fig. 1. In the base station, all channel information is sent to the sub-channel and power allocation algorithm through feedback channels from all mobile users. The resource allocation scheme made by the algorithm is forwarded to the OFDM transmitter. The transmitter then selects different numbers of bits from different users to form an OFDM symbol. The resource allocation scheme is updated as fast as the channel information is collected. In this paper, perfect instantaneous channel information is assumed to be available at the base station and only the broadcast scenario is studied. It is also assumed that the sub-channel and bit allocation information is sent to each user by a separate channel. Throughout this paper, we assume a total of K users in the system sharing N sub-channels with total transmit power constraint. Our objective is to optimize the sub-channel and power allocation in order to achieve the highest sum error-free capacity under the total power constraint. We use the equally weighted sum capacity as the objective function, but we introduce the idea of proportional fairness into the system by adding a set of nonlinear constraints. The benefit of introducing proportional fairness into the system is that we can explicitly control the capacity ratios among users, and generally ensure that each user is able to meet his target data rate, given sufficient total available transmit power.

Mathematically, the optimization problem considered in this paper is formulated as

$$\min_{p_{k,n}, \rho_{k,n}} \sum_{k=1}^K \sum_{n=1}^N \frac{p_{k,n}}{N} \log_2 \left(1 + \frac{p_{k,n} h_{k,n}^2}{N_0 \frac{B}{N}} \right)$$

Subject to $\sum_{k=1}^K \sum_{n=1}^N p_{k,n} \leq P_{total}$

$$p_{k,n} \in (0, 1] \text{ for all } k, n$$

$$\sum_{k=1}^K \rho_{k,n} = 1 \text{ for all } n$$

$$R_1, R_2, \dots, R_K = \gamma_1, \gamma_2, \dots, \gamma_K$$

Where K is the total number of users; N is the total number of sub channels; N0 is the power spectral density of additive white Gaussian noise; B and are the total available bandwidth and power,

respectively; is the power allocated for user k in the sub-channel

ρ_{k,n}; is the channel gain for user k in sub-channel n; ρ_{k,n} can only be the value of either 1 or 0, indicating whether sub-channel n is used by user k or not. The fourth constraint shows that each sub-channel can only be used by one user. The Capacity for user k, denoted as R_k, is defined as

$$R_k = \sum_{n=1}^N \frac{p_{k,n}}{N} \log_2 \left(1 + \frac{p_{k,n} h_{k,n}^2}{N_0 \frac{B}{N}} \right)$$

Finally, $\gamma_i, i=1, \dots, K$ is a set of predetermined values which are used to ensure proportional fairness among users.

III. OPTIMAL SUB CHANNEL ALLOCATION AND POWER DISTRIBUTION

The optimization problem in (1) is generally very hard to solve. It involves both continuous variables p_{k,n} and binary variables ρ_{k,n}. Such an optimization problem is called a mixed binary integer programming problem. Furthermore, the nonlinear constraints in (1) increase the difficulty in finding the optimal solution because the feasible set is not convex. In a system with K

users and N sub channels, there are possible sub channel allocations, since it is assumed that no sub-channel can be used by more than one user. For a certain sub-channel allocation, an optimal power distribution can be used to maximize the sum capacity, while maintaining proportional fairness. The optimal power distribution method is derived in the next section. The maximum capacity over all sub channel allocation schemes is the global maximum and the corresponding sub channel allocation and power distribution is the optimal resource allocation scheme. However, it is prohibitive to find the global optimizer in terms of computational complexity. A suboptimal algorithm is derived in this paper to reduce the complexity significantly while still delivering performance close to the global optimum. An alternative approach to make the optimization problem in (1) easier to solve is to relax the constraint that sub-channels can only be used by one user. Thus ρ_{k,n} is reinterpreted as the sharing factor of user k to sub-channel n, which can be any value on the half-open interval of (0; 1]. The optimization in (1) can be transformed into

$$\min_{p_{k,n}, \rho_{k,n}} \sum_{k=1}^K \sum_{n=1}^N \frac{p_{k,n}}{N} \log_2 \left(1 + \frac{p_{k,n} h_{k,n}^2}{N_0 \frac{B}{N}} \right)$$

Subject to $\sum_{k=1}^K \sum_{n=1}^N p_{k,n} \leq P_{total}$

$$p_{k,n} \geq 0 \text{ for all } k, n$$

$$\rho_{k,n} \in (0, 1] \text{ for all } k, n$$

$$\sum_{k=1}^K \rho_{k,n} = 1 \text{ for all } n$$

$$R_1, R_2, \dots, R_K = \gamma_1, \gamma_2, \dots, \gamma_K$$

That is, the original maximization problem is transformed into a minimization problem. In the third constraint in (4),

ρ_{k,n} is not allowed to be zero since the objective function is not defined for ρ_{k,n} = 0. However, when ρ_{k,n} is arbitrarily close to 0,

$$\log_2 \left(1 + \frac{p_{k,n} h_{k,n}^2}{\rho_{k,n} N_0 \frac{B}{N}} \right)$$

also approaches 0. Thus, the nature of the objective function remains unchanged by excluding the case $\frac{1}{2}k;n = 0$. A desirable property of the objective function in (4) is that it is convex on the set defined by the first two constraints. The convexity is shown in Appendix I. However, the nonlinear equality constraints make the feasible set non-convex. In general, such optimization problems require linearization of the nonlinear constraints. The linearization procedure may lead the solution slightly off the feasible set defined by the nonlinear constraints. There is always a trade off between satisfaction of the constraints and improvement of the objective. Furthermore, it is still computationally complex to find the optimal solution. For these reasons, we propose a sub optimal technique in the next section.

IV. SUBOPTIMAL SUB-CHANNEL ALLOCATION AND POWER DISTRIBUTION

Ideally, sub channels and power should be allocated jointly to achieve the optimal solution in (1). However, this poses a prohibitive computational burden at the base station in order to reach the optimal allocation. Furthermore, the base station has to rapidly compute the optimal sub-channel and power allocation as the wireless channel changes. Hence low- complexity sub optimal algorithms are preferred for cost effective and delay-sensitive implementations. Separating the sub channel and power allocation

$$\max_{p_{k,n}} \sum_{k=1}^K \sum_{n \in \Omega_k} \frac{1}{N} \log_2 \left(1 + \frac{p_{k,n} h_{k,n}^2}{N_0 \frac{E}{N}} \right)$$

is a way to reduce the complexity because the number of variables in the objective function is almost reduced by half. Section IV-A discusses a sub channel allocation scheme. Section IV-B presents the optimal power distribution given a certain sub channel allocation.

A. Suboptimal Sub-channel Allocation

In this section, we discuss a suboptimal sub channel algorithm based on [7]. In the sub optimal sub channel allocation algorithm, equal power distribution is assumed across all sub channels. We

define γ_k as the channel-to-noise ratio for user k in sub-channel n and Ω_k is the set of sub channels assigned to user

k. The algorithm can be described as

1) Initialization Set $\gamma_k = 0$,

$A = \{1, 2, \dots, N\}$

2) For $k=1$ to K

a) Find n satisfying $\gamma_k \geq \gamma_j$ for all $j \in A$

b) Let $\gamma_k = \gamma_n$, $A = A - \{n\}$ and update according to

3) While $A \neq \emptyset$

a) find k satisfying $\gamma_k \leq \gamma_j$ for all $i, 1 \leq i \leq k$

b) for the found k, find n satisfying $\gamma_k \geq \gamma_j$ for all $j \in A$

c) for the found k and n, let

$\gamma_k = \gamma_n$, $A = A - \{n\}$ and update according to (2)

The principle of the suboptimal sub-channel algorithm is for each user to use the sub channels with high channel-to- noise ratio as much as possible .At each iteration, the user with the lowest proportional capacity has the option to pick which sub-channel to use. The sub-channel allocation algorithm is suboptimal because equal power distribution in all sub channels is assumed. After sub-channel allocation, only coarse proportional fairness is achieved. The goal of maximizing the sum capacity while maintaining proportional fairness is achieved by the power allocation in the next section.

B. Optimal Power Distribution for a Fixed Sub-channel Allocation

To a certain determined sub-channel allocation, the optimization problem is formulated as

$$\begin{aligned} & \sum_{k=1}^K \sum_{n \in \Omega_k} p_{k,n} \leq P_{total} \\ & p_{k,n} \geq 0 \\ & \Omega_k \\ & \Omega_1 \quad \Omega_2 \quad \dots \quad \Omega_K \\ & R_1 \quad R_2 \quad \dots \quad R_K \quad \gamma_1 \quad \gamma_2 \end{aligned}$$

Where Ω_k is the set of sub-channels for user k, and Ω_k and Ω_l are mutually exclusive when $k \neq l$. the optimization problem in (5) is equivalent to finding the maximum of the following cost function L=

$$\begin{aligned} & \sum_{k=1}^K \sum_{n \in \Omega_k} \frac{1}{N} \log_2 (1 + p_{k,n} H_{k,n}) + \\ & \lambda_1 (\sum_{k=1}^K \sum_{n \in \Omega_k} p_{k,n} - P_{total}) + \\ & \sum_{k=2}^K \lambda_k (\sum_{n \in \Omega_k} \frac{1}{N} \log_2 (1 + p_{1,n} H_{1,n}) - \\ & \frac{\gamma_1}{\gamma_k}) \sum_{n \in \Omega_k} \frac{1}{N} \log_2 (1 + p_{k,n} H_{k,n}) \end{aligned}$$

$$\frac{1}{\gamma_k N_k} \left(\log_2 \left(1 + H_{k,1} \frac{P_{k,tot} - V_k}{N_k} \right) \right)$$

Where the lagrangian multipliers we differentiate (6) with respect to and set each derivative 0 to obtain

$$\frac{\partial L}{\partial p_{1,n}} = \frac{1}{N \ln 2} \frac{H_{1,n}}{1 + H_{1,n} P_{1,n}} + \lambda_1 + \sum_{k=2}^K \lambda_k \frac{1}{N \ln 2} \frac{H_{1,n}}{1 + H_{1,n} P_{1,n}} = 0$$

$$\frac{\partial L}{\partial p_{k,n}} = \frac{1}{N \ln 2} \frac{H_{k,n}}{1 + H_{k,n} P_{k,n}} + \lambda_1 - \lambda_k + \frac{\gamma_k}{\gamma_k N \ln 2} \frac{1}{1 + H_{k,n} P_{k,n}} = 0$$

For k=2,3,...,K and n ∈

1) Power Distribution for a single user: In this section the optimal power distribution strategy for a single user k is derived

From either (7) Or (8), we may obtain

$$\frac{H_{k,m}}{1 + H_{k,m} P_{k,m}} = \frac{H_{k,n}}{1 + H_{k,n} P_{k,n}}$$

For m,n ∈ and k=1,2,...,K. without loss of generality we assume that ≤ ≤ ... for k=1,2,...,K and is number of sub-channels in Thus,(9) can be rewritten as

$$P_{k,m} P_{k,1} = 10$$

For n=1,2,..., and k=1,2,...,K.equation (10) shows that the power distribution for a single user k on sub-channel n.more power will be put into the sub-channels with higher channel to noise ratio yhis is the water filling algorithm [13]in frequency domain by defining

as the total power allocated for user k and using (10), can be expressed as

$$P_{k,tot} = \sum_{n=1}^{N_k} P_{k,n} = N_k P_{k,1} + \sum_{n=2}^{N_k} P_{k,n} \quad (11)$$

2) Power Distribution among users: once the set is known power allocation can be determined by (10) and (11). The total power constraint and capacity ratio constraints in (5) are used to obtain

With (9) and (11),the capacity ratio constraints can be expressed as

$$\frac{1}{\gamma_1 N_1} \left(\log_2 \left(1 + H_{1,1} \frac{P_{1,tot} - V_1}{N_1} \right) \right) \log_2 w_1$$

log₂ w_k for

k=2,3,...,k, where and are defined as

$$V_k$$

(There are K variables in the set of K uations in (12)and(15).Solving the set of nctions provides the optimal power allocation heme. The equations are, in general, non linear methods, such as the Newton- Raphson -Newton methods [15], can be used to

the solution, with a certain amount of computational effort. In the Newton-Raphson method, the computational complexity primarily comes from finding the update direction. In Appendix II the computational complexity of each iteration is shown to be O(K).Under certain conditions ,the optimal or near-optimal solution to the set of nonlinear equations can be found in one iteration. Two special cases are analyzed below

Linear case

If : :.....: =: then the set of equations ,i.e.(12) and (15),can be transformed in to a set of linear equations with the following expression

$$\begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & a_{2,2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \dots & a_{K,K} \end{bmatrix} \begin{bmatrix} P_{1,tot} \\ P_{2,tot} \\ \vdots \\ P_{K,tot} \end{bmatrix} = \begin{bmatrix} P_{total} \\ b_2 \\ \vdots \\ b_K \end{bmatrix}$$

For k= 2,3,...,K. the matrix of (16) has nonzero elements only on the first .row, the first column and the main diagonal. By substitution, the solution to (16) can be obtained with a computational complexity of O(K).

High Channel-to-Noise Ratio Case

In adaptive modulation, the linear condition rarely happens and the set of equations remains nonlinear, which requires considerably more computation to solve. However, if the channel to-noise ratio is high, approximations can be made to simplify the problem First consider (13), in which V_k could be relatively small compared

$$\{P_{k,tot}\}_{k=1}^K$$

to if the channel to-noise ratios are high. Furthermore, if adaptive sub-channel allocation is used, the best sub-channels will be chosen and they have relatively small channel gain differences among them. Thus, the first approximations $V_k = 0$. Second, assuming that the base station could provide a large amount of power and the channel to noise ratio is high the term γ_k is much larger than 1 with the above two approximations (12) can be rearranged and simplified to be

$$\left(\frac{H_{k,1}W_k}{\gamma_k}\right)^{\frac{N_k}{\gamma_k}} (P_{tot})^{\frac{N_k}{\gamma_k}} \left(\frac{H_{k,1}W_k}{\gamma_k}\right)^{\frac{N_k}{\gamma_k}} (P_{tot})^{\frac{N_k}{\gamma_k}}$$

Substituting (19) into (15), a single equation with the variable P_{tot} can be derived as

$$\sum_{k=1}^K c_k (P_{tot})^{\alpha_k} P_{total}$$

Numerical algorithms, such as Newton's root-finding method

[14] or the false position method [14], can be applied to find the zero of (20).

C. Existence of Power Allocation Scheme

1) Solution to Single User Power Allocation:

For a certain user k , there is no power allocation if $\gamma_k < 1$. This situation could happen when a sub-channel is allocated to a user who does not have a high channel gain in that sub-channel. The greedy water-filling algorithm would rather stop using this sub-channel. In case this situation happens, the set of γ_k , as well as the corresponding values of N_k , V_k and W_k , need to be updated and the power allocation algorithm presented in this paper should be executed again, as shown in Fig. 2.

2) Solution to Multiuser Power Allocation:

In case that the channel-to-noise ratio is high, there is one and only one solution to (20) since every item in the summation monotonically increases and (20) achieves different signs at

$\gamma_k = 1$ and $\gamma_k = A$. A numerical algorithm can be used to find the solution to (20). The complexity of finding the solution will primarily rely on the choice of the numerical algorithm and the precision required in the results. After γ_k is found, P_{tot} can be calculated using (19). Then the overall power allocation scheme can be determined by (10) and (11). In general, it can be proved that there must be an optimal sub-channel and power allocation scheme that satisfies the proportional

fairness constraints and the total power constraint. Further more the optimal scheme must utilize all available power. Several facts lead to the above conclusion. First, to a certain user, the capacity of the user is maximized if water-filling algorithm is adopted. Furthermore, the capacity function is continuous with respect to the total

available power to that user. In other words, is continuous with P_{tot} . Second, if the

optimal allocation scheme does not use all available transmit power, there is always a way to redistribute the unused power among users while maintaining the capacity ratio constraints since I_k

is continuous with P_{tot} for all k . Thus, the Sum capacity is further increased. In Appendix II, we describe the Newton-Raphson method to find P_{tot} without considering the constraints on γ_k .

$\gamma_k > 1$ for $k = 1; 2; \dots; K$. If the Newton-Raphson method returns a non-feasible the set γ_k and the associated N_k , V_k , and W_k would need to be updated. The Newton-Raphson method should be performed until all

D. Complexity Analysis.

The best sub-channel allocation scheme can be found by exhaustive search; i.e., for each sub-channel allocation, one would run the optimal power allocation algorithm in Fig. 2, which has the computational complexity of $O(K)$. The subcarrier allocation that gives the highest sum capacity is the optimum. In a K -user N -sub-channel system, it is prohibitive to find the global optimum since there are

possible sub-channel allocations. The complexity of the proposed algorithm consists of two parts: sub-channel allocation with the complexity of $O(KN)$ and power allocation of $O(K)$. Hence the complexity of the proposed method is approximately on the order of KN times less than that of the optimal, because the power allocation is only executed once.

V. SIMULATION RESULTS

The simulation parameters considered are as follows: frequency selective multipath channel modeled as consisting of six independent Rayleigh multipaths. A maximum delay spread of $5\mu s$ and maximum Doppler spread of 30Hz

are considered. The channel information is sampled every 0.5ms to update the sub channel and power allocation. The total power is assumed to be 1w, the total bandwidth as 1 MHz, total carriers to be 64 and the average sub-channel SNR is assumed to be 38dB.

The simulation results show that among the different linear and non linear techniques being compared in the paper, the proposed linear method is less complex and the average cpu time required to perform the resource allocation is faar less in the case of linear technique than the root finding technique. Figure 4 shows the comparison of the proposed method with the root finding method and clearly the porposed method has a better capacity as the number of users increases which is an obvious case in the multi user environment.

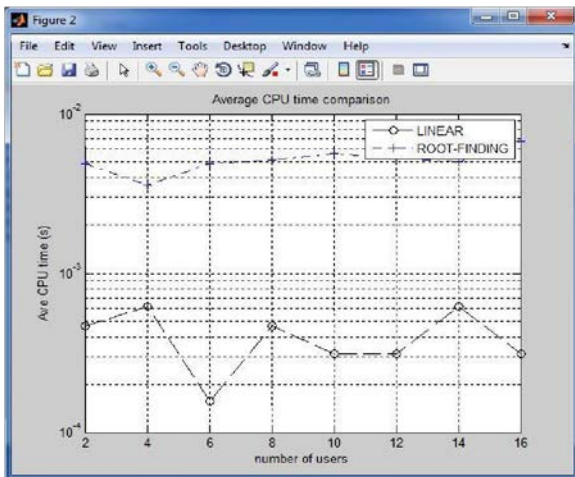


Fig. 2: Total Number of users Versus Avg CPU time

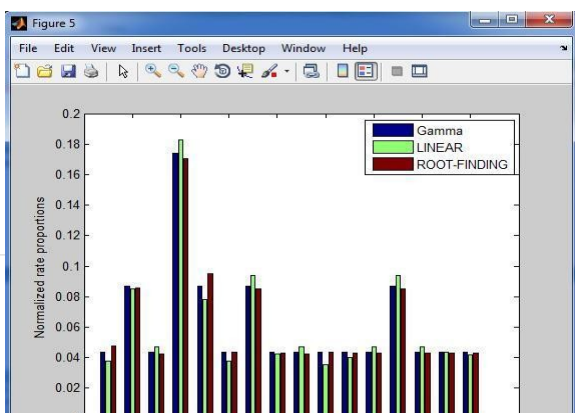


Fig. 3: Normalized rate proportions Vs Number of users for gamma, Linear and Root finding Method

VI. CONCLUSION

This paper provides a novel solution for the rate adaptive resource allocation problem with proportional rate constraints. It is shown through the results in this paper that the current work performs better than the previous work in this area in achieving higher total capacity while reducing the computational complexity to a great extent.

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